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BUCKLING OF ELASTIC COMPOSITE RINGS UNDER INTERNAL IMPULSIVE LOADING

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The experimentally detected phenomenon of compression fracture of a composite ring made of unidirectional fiberglass plastic under initial internal impulsive (explosive) loading is analyzed. Fracture results from bending in the compression phase because of loss of stability of the radial axisymmetric mode of motion.

Introduction. The loss of stability of the radial axisymmetric vibration mode of cylindrical shells or rings and transition to nonaxisymmetric flexural modes of motion accompanied by an exponential increase in the amplitudes of normal deflections has been the subject of extensive theoretical and experimental investigations [1–6]. This deformation and fracture pattern is observed for a thin-walled shell or a ring under sudden loading by an external radial pressure [1, 3, 5, 6]. However, for shells or rings made of composites, this fracture mechanism is also possible under internal impulsive loading [7–10]. This phenomenon is explained by the fact that composites, in particular unidirectional composites, in contrast to metals, have much larger limiting elastic strains for stretching in the direction of reinforcement (0.04–0.05 and 0.002–0.005, respectively) with almost complete absence of plastic strains until fracture [11]. During impulsive stretching of a composite shell or a ring in the radial direction, this makes it possible to accumulate a sufficient amount of elastic energy to ensure bending fracture of the material in the compression phase due to loss of stability of radial axisymmetric vibrations and transition to nonaxisymmetric flexural vibrations. One additional factor (along with initial irregularities and nonuniformity in the application of initial loads and distribution of initial velocities) that is responsible for loss of stability and fracture of this type for an elastic shell or a ring is the low (as compared to metals) shearing rigidity of the packet of reinforcing fibers in the plane of the ring.

In the present paper, we consider an elastic uniform (but not isotropic) cylindrical ring that models a ring made of a unidirectional composite reinforced with a fiber in the circumferential direction with zero angle of reinforcement. In this case, the composite material is considered transversely isotropic and is described by five independent elastic constants [12]. Since the ring deforms in its plane, only two of the elastic constants are used: the modulus of longitudinal elasticity E in the circumferential direction of reinforcement and the modulus of transverse shear G in the ring plane.

1. Derivation of the Equations of Motion of an Elastic Ring Taking into Account Transverse Shear Deformations. We convert the system of differential equations of equilibrium for a cylindrical shell [13] to a system of differential equations of equilibrium for a ring of radius R. For this, we eliminate the longitudinal coordinate α and the corresponding longitudinal displacement u, the axial and transverse forces N_1 and Q_1, the moments M_{12} and M_{21}, and the surface forces X. In addition, we redenote the derivative with respect to the coordinate $\partial/\partial\beta$ by $\partial/\partial\varphi$. Substituting into this system the expressions for the Y and Z components of the surface forces and the additional loads caused by the action of the axial

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forces N_1 and N_2 [14] and omitting the subscript 2, we obtain

$$\frac{\partial N}{\partial \varphi}+Q-Rm\frac{\partial^2 v}{\partial t^2}=0,\qquad -N+\frac{\partial Q}{\partial \varphi}-\frac{N}{R^2}\Big(\frac{\partial^2 w}{\partial \varphi^2}+w\Big)-Rm\frac{\partial^2 w}{\partial t^2}=0,$$
$$-\frac{\partial M}{\partial \varphi}-RQ=0, \tag{1}$$

where $m\partial^2 v/\partial t^2$ and $m\partial^2 w/\partial t^2$ are inertial forces.

If the Kirchhoff–Love hypotheses are used, the transverse force in the third equation of system (1) is expressed in terms of the bending moment, which, in turn, is expressed in terms of the elastic constants and displacement w (deflection). In this case, however, shear strain is ignored. As compared to metals, composites have low shear rigidity and similar (and sometimes even higher) rigidity in extension–compression. Therefore, for a composite shell or a ring, it is necessary to allow for the effect of the shearing strain [15]. For this, it is possible to use, for example, the Timoshenko hypothesis [16]. We then assume that $\theta=\psi+\gamma$, where θ is the angle of rotation of the normal to the bent axis of the ring, ψ is the angle of rotation of the rectilinear element, and γ is the angle of rotation of the rectilinear element about the normal (the angle of transverse shear). From [16, 17], we have the following expressions for θ, M, and Q:

$$\theta=\frac{1}{R}\Big(v-\frac{\partial w}{\partial \varphi}\Big),\quad M=-\frac{Eh^3}{12R(1-\nu^2)}\frac{\partial \psi}{\partial \varphi},\quad Q=kGh\gamma=kGh\Big[\frac{1}{R}\Big(v-\frac{\partial w}{\partial \varphi}\Big)-\psi\Big].$$

Here E is the modulus of elasticity of the material in extension–compression in the circumferential direction, h is the wall thickness of the ring, G is the modulus of transverse shears, and k is a constant factor that depends on the type of distribution function of tangential stresses across the thickness of the section. For a parabolic distribution of tangential stresses, this factor is $k=5/6$, and for a uniform distribution of tangential stresses across the thickness of the shell or ring, $k=1$ [18]. Substituting the expressions for M and Q into system (1) and introducing the notation $\delta=h^2/[12R^2(1-\nu^2)]^{-1}$, we obtain

$$\frac{\partial N}{\partial \varphi}+kGh\Big[\frac{1}{R}\Big(v-\frac{\partial w}{\partial \varphi}\Big)-\psi\Big]-Rm\frac{\partial^2 v}{\partial t^2}=0,$$
$$-N+kGh\Big[\frac{1}{R}\Big(\frac{\partial v}{\partial \varphi}-\frac{\partial^2 w}{\partial \varphi^2}\Big)-\frac{\partial \psi}{\partial \varphi}\Big]-\frac{N}{R^2}\Big(\frac{\partial^2 w}{\partial \varphi^2}+w\Big)-Rm\frac{\partial^2 w}{\partial t^2}=0, \tag{2}$$
$$Eh\delta\frac{\partial^2 \psi}{\partial \varphi^2}-kGh\Big[\frac{1}{R}\Big(v-\frac{\partial w}{\partial \varphi}\Big)-\psi\Big]=0.$$

In this case, we have a system of three differential equations for three independent unknown functions (v is the displacement along the circumferential coordinate, w is the normal deflection, and ψ is the angle of transverse shear. For isotropic materials, systems of equations of motion of this type for thin shells taking into account transverse shear strain are given in [18], and for composites, they are presented in [19, 20]. In addition, the last reference contains the most complete bibliography on this problem.

Next, system (2) is converted so that it reduces to one equation. As in [2–4], we assume that the interaction of circumferential stresses of the axisymmetric mode of motion with bending can cause strong growth of only those bending strains for which the median line of the ring appears inextensible. Then, from the condition of equality to zero of the circumferential strain, $\varepsilon_\varphi=(1/R)(\partial v/\partial\varphi+w)=0$, we obtain $\partial v/\partial\varphi=-w$ and $\partial^3 v/\partial\varphi^3=-\partial^2 w/\partial\varphi^2$. In view of this, after simple transformations system (2) reduces to one equation for the function w:

$$\frac{D}{R^3}\Big(\frac{\partial^6 w}{\partial \varphi^6}+2\frac{\partial^4 w}{\partial \varphi^4}+\frac{\partial^2 w}{\partial \varphi^2}\Big)+\frac{N}{R}\Big[\frac{E\delta}{G}\frac{\partial^6 w}{\partial \varphi^6}+\Big(1+\frac{E\delta}{G}\Big)\frac{\partial^4 w}{\partial \varphi^4}+\frac{\partial^2 w}{\partial \varphi^2}\Big]$$
$$+Rh\rho\frac{\partial^2}{\partial t^2}\Big[\frac{E\delta}{G}\frac{\partial^4 w}{\partial \varphi^4}+\Big(1-\frac{E\delta}{G}\Big)\frac{\partial^2 w}{\partial \varphi^2}-w\Big]=0. \tag{3}$$

Here $D=Eh^3/(12(1-\nu^2))$ is the cylindrical rigidity, $\delta=h^2/(12R^2(1-\nu^2))$, and $m=\rho h$.

The above equation is a differential equation of flexural vibrations of a circular ring in its plane that takes into account transverse shear strains and ignores the rotational inertia of the cross section. The terms of the working equation (3) that contain the factors $E\delta/G$ can exert a significant effect on the results of solution of both static and dynamic problems. For the case where shear is ignored, it is possible to pass to the limit $G \to \infty$. Thus, Eq. (3) becomes an equation of flexural vibrations of the ring in its plane that coincides with the equations given in [4, 21].

Below, we analysis of Eq. (3) in two cases:

— the axial compressing force is generated by an external, instantaneously applied hydrostatic pressure that remains constant during deformation;

— the axial load is a time-dependent potential function, and the initial axisymmetric deformation of the ring is produced by a velocity that is instantaneously applied at the initial time or by instantaneously removed constraints that were imposed previously.

2. Buckling of a Ring under Impulsive Loading. We examine the stability of the flexural motions of the ring under the action of instantaneously applied external pressure p that exceeds the critical value. Taking into account that the normal deflection function $w(\varphi)$ is periodic along the circumferential coordinate in the case of flexural vibrations and that $N = -pR$, we seek a solution in the form

$$w = \sum_{n=1}^{\infty} q_n(t) \sin(n\varphi), \tag{4}$$

where $q_n(t)$ are functions of time for the amplitudes of the corresponding motions. Substituting expression (4) into Eq. (3), for the function $q_n(t)$, we obtain the ordinary differential equation

$$\frac{\partial^2 q_n}{\partial t^2}\left[\frac{E\delta}{G}n^4 + \left(1+\frac{E\delta}{G}\right)n^2 + 1\right] + q_n\left\{\frac{D}{R^4 h\rho}[-n^6 + 2n^4 - n^2] + \frac{p}{Rh\rho}\left[\frac{E\delta}{G}n^6 + \left(1-\frac{E\delta}{G}\right)n^4 - n^2\right]\right\} = 0. \tag{5}$$

For the case of static external pressure p, the critical external pressure is

$$p^* = \frac{D}{R^3}\frac{n^6 - 2n^4 + n^2}{(E\delta/G)n^6 + (1-(E\delta/G))n^4 - n^2} = \frac{D}{R^3}\frac{n^2-1}{1+(E\delta/G)n^2}.$$

With allowance for transverse shear, the least Euler load (at $n = 2$) is

$$p_e^* = \frac{D}{R^3}\frac{3}{1+4E\delta/G}.$$

This formula for the least critical load coincides with the expression of the critical load derived in [22] for a circumferential arc with a cone angle of 180° taking into account the effect of the transverse force (shear). According to the Lavrent'ev–Ishlinskii approach described in [1], the phenomenon of loss of stability of the radial axisymmetric mode of motion and transition to nonaxisymmetric flexural modes of motion is treated as a change of the type of solution of the differential equation

$$\frac{\partial^2 q_n}{\partial t^2} + \frac{D}{R^4 h\rho}\frac{n^2(n^2-1)(n^2-\eta^2)}{n^2+1}q_n = \frac{f_n}{Rh\rho(n^2+1)}, \qquad \eta^2 = \frac{pR^3}{D} + 1,$$

which arises after substitution of expression (4) into the equation of small flexural vibrations of a ring in its plane under uniform external pressure p:

$$Rh\rho\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w}{\partial\varphi^2} - w\right) + \frac{D}{R^3}\left(\frac{\partial^6 w}{\partial\varphi^6} + 2\frac{\partial^4 w}{\partial\varphi^4} + \frac{\partial^2 w}{\partial\varphi^2}\right) + p\left(\frac{\partial^4 w}{\partial\varphi^4} + \frac{\partial^2 w}{\partial\varphi^2}\right) = f(\varphi),$$

where $f(\varphi)$ is a function determined by the initial geometric irregularities. In the case of positive values of the coefficient at q_n, the solution is a sum of harmonic functions and describes the vibrations of the system. In the case of negative values of the coefficient, the solution is the sum of two exponents, one of which damps and tends to zero, and the second increases without bound. This implies an exponential increase in the amplitudes

TABLE 1

n	p^0/p_e^0 (I)	p^*/p_e^* (II)	p^*/p_e^* (III)	p^*/p_e^* (IV)	n^*
3	2.7	2.5	2.4	2.6	2
4	5.0	4.5	4.1	4.9	3
5	8.0	6.7	5.9	7.7	4
6	11.7	9.1	7.5	11.1	4
7	16.0	11.5	9.1	15.1	5
8	21.0	13.7	10.4	19.3	6
9	26.7	15.8	11.5	24.0	6
10	33.0	17.8	12.5	29.0	7
11	40.0	19.6	13.3	34.1	8
12	47.7	21.3	14.1	39.6	9
13	56.0	22.8	14.7	45.2	9
14	65.0	24.0	15.1	50.8	10
15	74.7	25.3	15.7	56.5	11

Note. I) ignoring shear; II) $E/G = 10$ and $R/h = 10$; III) $E/G = 20$ and $R/h = 10$; IV) $E/G = 10$ and $R/h = 25$.

TABLE 2

n^*	ε_{pe}, %
2	0.73
3	1.37
4	2.19
5	3.12
6	4.37
7	5.60
8	—
9	—

Note. $R/h = 10$.

of normal deflections q_n and is understood as loss of stability of the system. In our case, we consider Eq. (5) with zero right side $f(\varphi) = 0$. The critical loads of the static problem ignoring shear are given by the formula $p^0 = D/R^3(n^2-1)$ $(k = 2, 3, 4, \ldots, n)$ the lowest of which at $n = 2$ (Euler load) is $p_e^0 = 3D/R^3$ (Moris Lévy's solution).

Thus, the coefficients α ignoring shear [1] and the coefficients α^* taking into account shear describe the rate of loss of stability for the dynamic mode with number n, i.e., the coefficients at the exponents are given by the formulas

$$\alpha = \sqrt{\frac{D}{R^4 h\rho}\,\frac{n^2(n^2-1)}{n^2+1}\left(3\,\frac{p}{p_e} + 1 - n^2\right)},$$

$$\alpha^* = \sqrt{\frac{D}{R^4 h\rho}\,\frac{n^2(n^2-1)}{(E\delta/G)n^4 + (1+E\delta/G)n^2 + 1}} \times \sqrt{\left[\frac{3}{1+4E\delta/G}\,\frac{p}{p_e} + 1 - n^2\left(1 - \frac{E\delta}{G}\,\frac{3}{1+4E\delta/G}\,\frac{p}{p_e}\right)\right]}.$$

A comparison of the expressions for the coefficient α with and without shear shows that in the last case, the value of the coefficient depends not only on the rigid and geometric characteristics but also on the value of the ratio $E\delta/G$. This ratio also determines the ratio of p^* to the corresponding (with allowance for shear) Euler load p_e^* at dynamic loads equal to the series values of the critical loads of the static problem. In this case, the largest values of the coefficient α are for harmonics with number $n = n^*$, which is determined as the nearest integer to the value of n as to the parameter for which the function $\alpha(n)$ reaches a maximum value, and the number n^* is the number (in our case, the number of flexural waves along the circle of the ring) of the most rapidly increasing buckling mode. Table 1 gives the calculated ratio of the critical load p^* to the Euler load p_e^* and n^* under dynamic loads equal to successively increasing values n of the critical loads of the static problem (i.e., the loads at which, in statics, n waves form along the circle of the ring) for various ratios E/G (from 10 to 20), which correspond to real values of the strength properties of high-strength unidirectional composites [23].

Analysis of the results given in Table 1 shows that allowance for transverse shear strains for flexural motions of composite rings in dynamics leads to a considerable decrease in the ratio of the critical compressing load to the static Euler load for higher-order buckling modes. Typically, the number of waves that form upon higher-order-mode buckling (the value of n^*) does not change and coincides with the values in [1], calculated ignoring the shear effect, and there is only a decrease in the magnitude of the critical load (the magnitude of overload as compared to static load) at which there is dynamic buckling for the mode considered. For a thin-wall ring loaded by internal or external hydrostatic pressures, the following relations hold:

$$\sigma = p\,\frac{R}{h}, \quad \varepsilon = \frac{\sigma}{E}, \quad p^* = \frac{D}{R^3}\,\frac{n^2-1}{1+(E\delta/G)n^2} \to \varepsilon = \frac{h^2(n^2-1)}{12R^2(1-\nu^2)[1+(E\delta/G)n^2]}.$$

Thus, to accumulate, during extension, elastic energy that is sufficient for compression failure, the extended ring should reach the elastic strain defined by the expression given above. Table 2 gives the circumferential strains that correspond to the applied external distributed pressure at which the ring under static loading would lose stability of higher modes with formation of more than two flexural waves along the circle (ignoring shear). Hence it follows that a ring with the indicated geometric parameters made of a metal or a composite with limiting elastic strains less than 1% cannot fail by this mechanism since it fails in tension before this. At the same time, formation of more than seven flexural waves along the circle even for a ring made of unidirectional fiberglass plastic is improbable because of the excess of the limiting failure strain for fiberglass.

3. Parametric Vibrations of a Ring. For the unperturbed axisymmetric vibration mode of a ring, the differential equation of motion is obtained from system (2), in which the only remaining displacement is the normal deflection w, which does not depend on the angle φ.

We examine the stability of the flexural motions of the ring under axial loading N which is periodic in time:

$$Rh\rho\,\frac{\partial^2}{\partial t^2}\Big[\frac{E\delta}{G}\,\frac{\partial^4 w}{\partial\varphi^4} + \Big(1-\frac{E\delta}{G}\Big)\frac{\partial^2 w}{\partial\varphi^2} - w\Big] + \frac{D}{R^3}\Big(\frac{\partial^6 w}{\partial\varphi^6} + 2\,\frac{\partial^4 w}{\partial\varphi^4} + \frac{\partial^2 w}{\partial\varphi^2}\Big)$$
$$+\frac{Eh\varepsilon_{\varphi 0}\cos\tau}{R(1-\nu^2)}\Big[\frac{E\delta}{G}\,\frac{\partial^6 w}{\partial\varphi^6} + \Big(1+\frac{E\delta}{G}\Big)\frac{\partial^4 w}{\partial\varphi^4} + \frac{\partial^2 w}{\partial\varphi^2}\Big] = 0. \tag{6}$$

We divide all terms of Eq. (6) by $Rh\rho$, introduce the dimensionless time $\tau = tc/R$, where $c^2 = E/[\rho(1-\nu^2)]$, and reduce the common multiplier. As in Sec. 3, we seek a solution in the form (4)

$$\frac{\partial^2 q_n}{\partial\tau^2}\Big[1+\Big(1-\frac{E\delta}{G}\Big)n^2 - \frac{E\delta}{G}n^4\Big]$$
$$+\Big[\alpha(n^6-2n^4+n^2) - \varepsilon_{\varphi 0}\cos\tau\Big(\Big(1+\frac{E\delta}{G}\Big)n^4 - \frac{E\delta}{G}n^6 - n^2\Big)\Big]q_n = 0.$$

In this case, for the functions $q_n(\tau)$, we obtain the following Mathieu differential equation with real coefficients [24]:

$$\ddot{q}_n + (\lambda_n - \mu_n\cos\tau)\,q_n = 0.$$

Here, in our case,

$$\lambda_n = \alpha\,\frac{n^6-2n^4+n^2}{1+(1-E\delta/G)n^2-(E\delta/G)n^4}, \quad \mu_n = \varepsilon_{\varphi 0}\,\frac{(1+E\delta/G)n^4-(E\delta/G)n^6-n^2}{1+(1-E\delta/G)n^2-(E\delta/G)n^4}. \tag{7}$$

From the solution of the Mathieu equation it follows that the amplitudes $q_n(\tau)$ can increase without bound if the point with the coordinates (λ_n, μ_n) is in an instability region of a Mathieu diagram [24]. This diagram is shown schematically in Fig. 1a, where the dashed regions are stability regions. The solid curves in Fig. 1b show a Mathieu diagram for small values of (λ_n, μ_n). To determine which mode numbers n fall in the instability region, it is necessary to plot, on the Mathieu diagram, points whose coordinates are calculated from formulas (7) for integer values of n ($n = 2, 3, 4, 5, 6, \ldots$). In this case,

$$\mu_n = \varepsilon_{\varphi 0}\,\frac{(1+E\delta/G)n^4-(E\delta/G)n^6-n^2}{1+(1-E\delta/G)n^2-(E\delta/G)n^4} = \varepsilon_{\varphi 0}\,\frac{n^4-n^2}{n^2+1}.$$

Thus, the value of μ_n depends not on the value of the ratio $E\delta/G$ but only on the value of the initial specified elastic strain (or the instantaneously applied initial velocity).

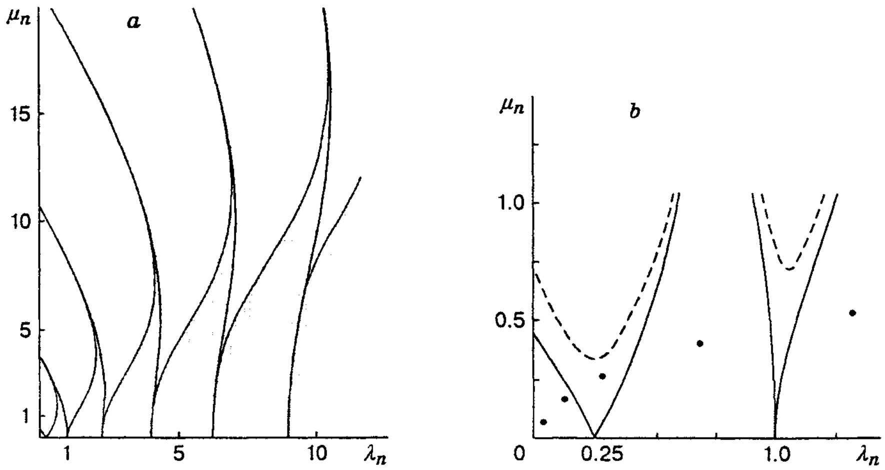

Fig. 1

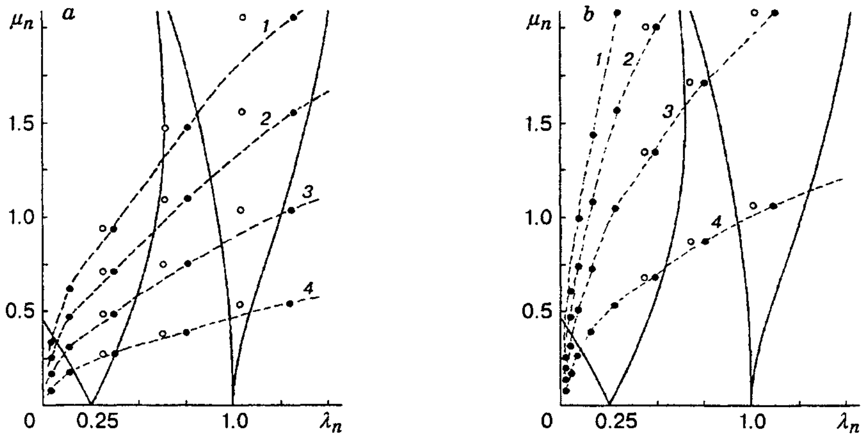

Fig. 2

For real high-strength unidirectional composites (including fiberglass), the ratio of the elastic constants E/G can be large enough: 10–20 and higher [23]. Therefore, in each particular case, a system can fall in both the dynamic instability region and the dynamic stability regions. Figure 2 shows calculated points that correspond to the numbers $n = 3, 4, \ldots, 10$ for various geometric and rigid ratios of the elastic ring with $R/h = 10$ (Fig. 2a) and $R/h = 25$ (Fig. 2b). Filled points correspond to $E/G = 10$, and open points to $E/G = 20$. The initial strain was limited by a value of 0.04, and the dashed curves 1–4 correspond to initial tensile strains $\varepsilon_{\varphi 0} = 4$, 3, 2, and 1%. Vibration modes with n larger than 10 were not considered since, with increase in the parameter λ_n, the dynamic stability regions in the Mathieu diagram merge (see Fig. 1a), and the probability of the system falling in the very narrow regions of dynamic instability is extremely low. It is obvious that at a certain level of initial strains, which determines the value of μ_n, some modes of flexural vibrations fall in the dynamic instability region. Although it is rather difficult to trace the dynamics of such process, one might expect that fracture occurs precisely for this mode. It should be noted that as the ring thickness decreases, the effect of the stiffness ratio E/G becomes negligible (Fig. 2b), and for thin rings and shells with ratio $R/h > 100$, it can be ignored altogether. For small (for composites) initial strains (about 0.001–0.005), dynamic instability does not develop even in the case of many tens of vibration periods because the value of μ_n is small in this case and the curve formed by points with mode numbers is close to the abscissa. In this case, because of the presence of internal damping in the system, the real Mathieu stability diagram lies somewhat higher and does not touch the horizontal axis [14] (dashed curves in Fig. 1b), and none of the modes of motion can fall in the instability region. This has been confirmed experimentally for some types of composites and metals [25, 26].

When more than one flexural vibration mode falls in the instability region at specified geometric and

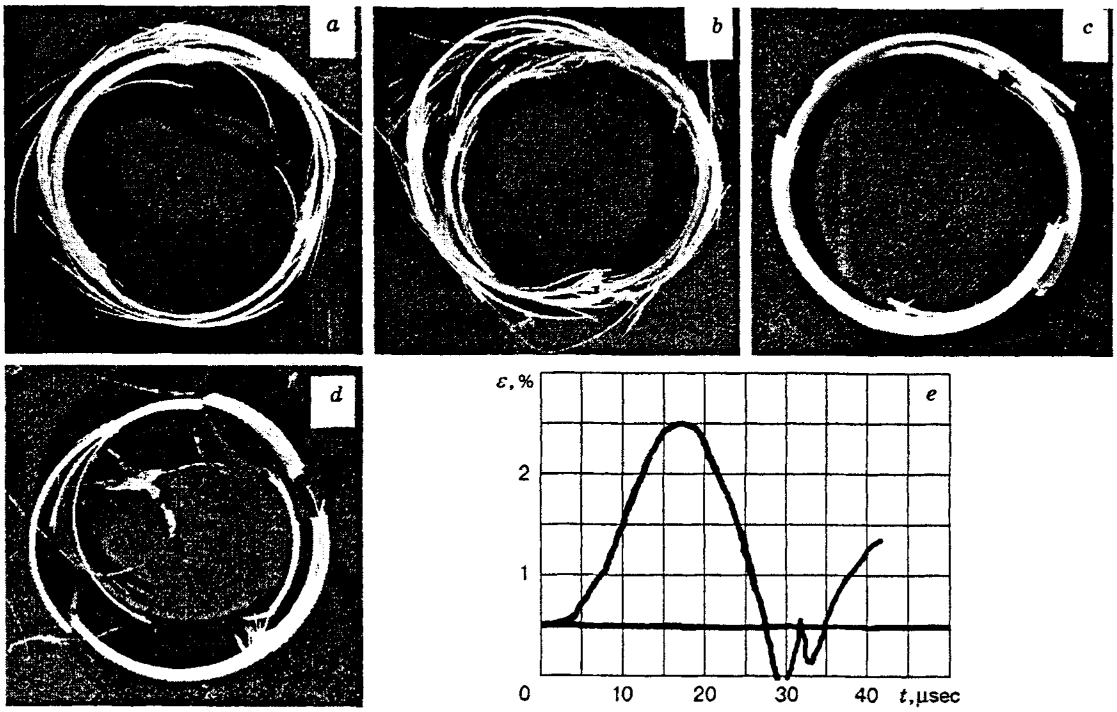

Fig. 3

rigid parameters, the present analysis cannot predict which of them dominates. In this case, it is necessary to employ numerical methods, similar to those described in [27, 28], to solve the equations of motion for an elastic ring or shell.

4. Comparison with Experimental Results. Internal impulsive loading of fiberglass rings (the reinforcing fibers are VM fiberglass and ÉDT-10 epoxy matrix) with wall thickness 1 to 5 mm was performed in accordance with the scheme of [26], shown in Fig. 1. The cylindrical explosive charge was a cast rod with diameter of 8 to 12 mm made of 50/50 TNT/RDX alloy. Figure 3a–d shows the appearance of ring-type fiberglass specimens that underwent bending fracture in compression under initial internal impulsive loading and a typical oscillogram of deformation and fracture of a tubular specimen made of the same material (Fig. 3e). The fracture of the ring-type and tubular specimens occurred in the compression phase after the first maximum of tensile circumferential deformation was reached. The fact that the specimens did not fail in tension was established from oscillograms of deformation, from which it followed that the limiting failure strains of the material were not reached in tension. Visual inspection of the loaded specimens showed that: (1) failure began on the inside of the ring or shell; (2) the external fibers of the material at the same places were not broken. Different numbers (three to six) of cracks, depending on the wall thickness of the ring-type specimen or shell and the value of the applied impulse (diameter of the explosive charge), were located periodically along the perimeter of the walls of the specimen. Specimens with three cracks are shown in Fig. 3a and b, specimens with four cracks are shown in Fig. 3c, and specimens with five cracks along the circle are shown in Fig 3d. It is remarkable that a similar fracture mechanism was observed only for fiberglass rings and shells [7–10] and was not observed for acrylic plastic, which possesses smaller limiting elastic failure strains and much higher damping ability [23].

5. Conclusions. A differential equation of flexural motion for an elastic ring is obtained and studied with allowance for the action of the transverse force, i.e., with allowance for transverse shear strains. This equation describes the behavior of an elastic high-strength composite ring with a high ratio of the longitudinal elasticity modulus to the transverse shear modulus due to the low general shear rigidity of the reinforcing fiber packet of the composite in the ring plane because of the low shear rigidity of the epoxy binder.

Analysis of the solution of the equation of flexural motion for the elastic ring shows that allowance for transverse shear strains in flexural motions of composite rings in dynamics leads to a considerable decrease in the ratio of the critical compressing load to the static buckling load for higher-order flexural modes.

Furthermore, analysis of the parametric flexural vibrations of a ring taking shear into account indicates that buckling is possible for the radial axisymmetric mode of motion and that the amplitudes of nonaxisymmetric flexural modes can increase. It is established that the value of the ratio of the elastic constants of the composite influences the possibility of the system falling in the dynamic instability region.

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